RQM exam April 9. 2025.

1a]
$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}m^2\phi^2 - \tfrac{1}{4!}\lambda\phi^4.$$
$$\frac{\delta \mathcal{L}}{\delta\phi} = -m^2\phi - \tfrac{1}{6}\lambda\phi^3$$
$$\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi} = \partial_\mu\phi$$
— 5pts.
$$\Rightarrow \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\right) - \frac{\delta\mathcal{L}}{\delta\phi} = \Box\phi + m^2\phi + \tfrac{1}{6}\lambda\phi^3 = 0.$$

1b]
$$T_{\mu\nu} = \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\cdot\partial_\nu\phi - \eta_{\mu\nu}\mathcal{L}$$
5pts.
$$= \partial_\mu\phi\cdot\partial_\nu\phi - \eta_{\mu\nu}\mathcal{L}$$
— 5pts
$$= \partial_\mu\phi\partial_\nu\phi - \tfrac{1}{2}\eta_{\mu\nu}(\partial\phi)^2 + \tfrac{1}{2}\eta_{\mu\nu}m^2\phi^2 + \tfrac{1}{4!}\eta_{\mu\nu}\lambda\phi^4$$
— 5pts.

1c]
$$[m] = 1$$
$$[\phi] = [\partial] = 1$$
— 5pts.
$$[\lambda] = 0$$
— 5pts.

2a]
$$H = \int d^3x\left(\tfrac{1}{2}\dot\phi^2 + \tfrac{1}{2}(\nabla\phi)^2 + \tfrac{1}{2}m^2\phi^2\right)$$
$$[H, \phi(\vec{x})] = \int d^3\vec{y}\left[\tfrac{1}{2}\pi(\vec{y})^2, \phi(\vec{x})\right]$$
5pts
$$= \tfrac{1}{2}\int d^3\vec{y}\left([\pi(\vec{y}), \phi(\vec{x})]\pi(\vec{y}) + \pi(\vec{y})[\pi(\vec{y}), \phi(\vec{x})]\right)$$
$$= -\tfrac{1}{2}i\left(\pi(\vec{x}) + \pi(\vec{x})\right) = -i\cdot\pi(\vec{x}).$$
— 5pts.

2b]
$$a_{\vec{p}_1} a^\dagger_{\vec{p}_2} a^\dagger_{\vec{p}_3} = a^\dagger_{\vec{p}_2} a^\dagger_{\vec{p}_3} a_{\vec{p}_1} + (2\pi)^3\cdot\left[\delta^{(3)}(\vec{p}_1 - \vec{p}_2)\cdot a^\dagger_{\vec{p}_3} + \delta^{(3)}(\vec{p}_1 - \vec{p}_3)\, a^\dagger_{\vec{p}_2}\right].$$
— 10pts.

2c]

Decays exponentially. — 5pts

this is a Lorentz-invariant expression, and hence observer-independent. — 5pts

3a]

$$\partial_\mu T^{\mu\nu} = \partial_\mu \left(i \bar{\psi} \gamma^\mu \partial^\nu \psi \right)$$

$$= i \cdot (\partial_\mu \bar{\psi}) \gamma^\mu \cdot \partial^\nu \psi + i \bar{\psi} \partial^\nu \not{\partial} \psi$$ — 5pts

[Dirac: $(i\not{\partial} - m)\psi = 0$

$\bar{\psi}(i\overleftarrow{\not{\partial}} + m) = 0$.]

$$= -m \bar{\psi} \gamma^\nu \psi + m \bar{\psi} \cdot \partial^\nu \psi = 0.$$ — 5pts

3b]

$$H \left| b^{s_1 +}_{\vec{p}_1} \cdot c^{s_2 +}_{\vec{p}_2} \right| 0 \rangle$$

$$= \left(E_{\vec{p}_1} + E_{\vec{p}_2} \right) \left(b^{s_1 +}_{\vec{p}_1} c^{s_2 +}_{\vec{p}_2} \right| 0 \rangle$$ — 5pts

$\Rightarrow$ eigenvalue $E_{\vec{p}_1} + E_{\vec{p}_2}$ — 5pts.

3c]

$$[S^{\mu\nu}, \{\gamma^\rho, \gamma^\sigma\}]$$

$$= 2 \cdot [S^{\mu\nu}, \eta^{\rho\sigma} \cdot \mathbb{1}_4] = 0.$$ — 5pts

as $\mathbb{1}_4$ commutes with all γ-matrices. — 5pts